



CIRCULAR INTEREST – INTEREST ON AVERAGE BALANCES

Introduction to Circular Interest

When authoring formulas in Excel, a user may often encounter a circular reference. In this article we demonstrate how to solve a common problem related to interest using high school mathematics rather than Visual Basic.

A circular reference is created when the formula is directly or indirectly dependent on itself. Circular Logic is when $C=A+B$ but in turn A or B is a function of C.

Although possible to solve using an iterative approach this breaks a fundamental rule in efficient financial modelling. The widely used solution in the market is strewn with shortcomings so here we demonstrate how we have solved this problem algebraically.

Application

A classic example is that of interest on a deposit or a loan account, where the interest is calculated on the average balance and the interest accumulates within the account, e.g. during construction period. It is fair to say that this simple situation has stumped the Project Finance industry for the past several decades.

It can readily be calculated in a spreadsheet by accommodating a Circular Reference however this action has several consequences:

- It breaks a firm rule of Best Practice Modelling
- It masks additional circular references if introduced
- Reputable model review firms will qualify their formal opinion
- It breaches the in-house modelling risk policies in reputable institutions

The problem can be solved crudely by isolating the circular reference using a copy-and-paste macro. This involves copying the calculated interest and pasting it into the account, where upon the interest calculation is updated and the loop happens once more.

This is performed until the difference between the “calculated interest” and the “value copied interest” is below a level of tolerance in all periods. As soon as the model parameters change this macro will need to be run again.

Drawbacks of Circular Models

This is the general practice in the market however it has the following additional drawbacks

- It cannot be used readily with “Data Tables” – a powerful scenario tool
- It relies on the user to know when and how to execute the macro
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- It relies on ‘solid’ VBA coding otherwise the cell references may move

Like many calculations found in Project Finance there is a simple way and a hard way. Reflecting on what one is really trying to solve usually leads to a more elegant analytical rather than iterative approach.

“Circular references are a problem in transaction modelling and occur easily in Project Finance due to the interdependency of cashflows. Most circular references can be solved with simple algebra or proper cell referencing. Only in very few instances is VBA required.

Nick Crawley, Managing Director
Navigator Project Finance

The Mathematics

A circular reference is formed when interest is a function of the average balance of the loan:

$$\text{Interest} = \frac{1}{2} * (\text{Opening Balance} + \text{Closing Balance}) * \text{Interest Rate}$$

The above formula can be rearranged by simultaneous substitution.

Let,

CB = Closing Balance

OB = Opening Balance

I = Interest Earned

r = Deposit Interest Rate

Then we have:

$$1. \text{CB} = \text{OB} + \text{I}$$

$$2. \text{I} = \frac{1}{2} * (\text{OB} + \text{OB} + \text{I}) * r$$

$$3. 2 * \text{I} = 2 * \text{OB} * r + \text{I} * r$$

$$4. \text{I} = 2 * \text{OB} * r / (2 - r)$$

Thus, the algebraic solution for interest is:

$$\text{Interest} = (2 * \text{Opening Balance} * \text{Interest Rate}) / (2 - \text{Interest Rate})$$

The Modelling

To demonstrate this method we have put a simple workbook example where the interest on the debt account is calculated on



the average balance and the interest accumulates within the debt account.

If you want to generate Data Table-like outputs in a model where VBA is used for optimization then you should read more about our VBA for Financiers course.

The team at Navigator Project Finance
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To illustrate the problem above, we solve the interest in this example both with a traditional 'copy-and-paste' routine and then algebraically. You will see they yield the same result.

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Copy-and-paste approach:

- Interest is firstly calculated based on the formula:

$$\text{Interest} = \frac{1}{2} * (\text{Opening Balance} + \text{Closing Balance}) * \text{Interest Rate}$$

E.g. in period ending 31-Dec-08:

$$I = \frac{1}{2} * (108,333 + 118,543) * 9\% = 10,209$$

- To avoid circular reference, the calculated interest needs to be copied and pasted using a macro. (The "copy-and-paste" macro button is built in the accompanying workbook.)
- The above is performed until the difference between the "calculated interest" and the "value copied interest" (or "Delta") is below a level of tolerance in all periods.

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The algebraic solution for interest is:

$$\text{Interest} = (2 * \text{Opening Balance} * \text{Interest Rate}) / (2 - \text{Interest Rate})$$

E.g. in period ending 31-Dec-08:

$$I = (2 * 108,333 * 9\%) / (2 - 9\%) = 10,209$$

Period Start		Jan-07	Jan-08	Jan-09	Jan-10
Period End	Start End	Dec-07	Dec-08	Dec-09	Dec-10
Construction	1-Jan-07 31-Dec-10				
Operations	1-Jan-11 31-Dec-17				
Tutorial Calculations					
Debt Account					
Balance B/f	AUD '000 Selected Method	100,000	108,333	118,543	128,421
Interest during Construction	AUD '000 Algebraic	8,333	10,209	9,879	10,702
Balance C/f	AUD '000	108,333	118,543	128,421	139,123
Interest: Construction	% p.a.	8.00%	9.00%	8.00%	8.00%
Interest Calc: Traditional Copy & Paste					
Interest Calculated	AUD '000 Copy & Paste (Approach 1)	8,333	10,209	9,879	10,702
Interest Applied	AUD '000	8,333	10,209	9,879	10,702
Delta	AUD '000	-	-	-	-
Macro has been run?	Yes				
Interest Calc: Algebraic					
Interest	AUD '000	8,333	10,209	9,879	10,702

Screenshot: Solving interest calculated on the average balance

The Benefits

As shown in the workbook example, the interest could be solved using the mathematics without the VBA macro. The algebraic solution has some benefits over the traditional approach:

- Using an algebraic solution is transparent and easier to follow
- Errors in the formula can be traced to the source rather than to values
- Not everyone can author or read VBA script— an algebraic formula can be created and tested by anyone familiar with Excel.

This is a Free Tutorial from Navigator Project Finance. If you have any feedback or suggestions for future developments we would like to hear from you!

The team at Navigator Project Finance
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About Navigator Project Finance

Founded in 2004, Navigator Project Finance Pty Ltd (Navigator) is the project finance modelling expert. Headquartered in Sydney, Australia, Navigator is raising the global benchmark in financial modelling services to the project finance sector. Navigator designs and constructs financial models for complex project financings, offers training courses throughout the Middle East, Asia and Europe, and conducts independent model reviews of project finance transaction models. Navigator delivers fast, flexible and rigorously-tested project finance services that provide unparalleled transparency and ease of use.

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